PERIODICAL ROOM
GENER... LIBRARY
UNIV. OF ...

AUG 17 1925

THE JOURNAL OF PHILOSOPHY

VOL. XXII. NO. 17: AUGUST 13, 1925

CONTENTS

Mathematics and Credulity: E. T. BELL 449

Classification and Division: A. CORNELIUS BENJAMIN 458

Book Reviews:

University of California Publications in Philosophy, Volume IV and Volume V: E. A. SINGER, JR. 464

Fung's A Comparative Study of Life Ideals: K. S. LATOURETTE 473

Journals and New Books 474

Notes and News 476

PUBLISHED FORTNIGHTLY
BY THE
JOURNAL OF PHILOSOPHY, INC.

THE JOURNAL OF PHILOSOPHY

All communications to the Editors should be addressed to Professor FREDERICK J. E. WOODBRIDGE, or Professor WENDELL T. BUSH, or Professor HERBERT W. SCHNEIDER, Columbia University, New York.

Subscriptions and advertisements should be sent to THE JOURNAL OF PHILOSOPHY, 515 West 116th Street, New York. The annual subscription price (twenty-six numbers) is $4.00 and the charge for single numbers is 20 cents. Advertising rates will be forwarded on application.

THE JOURNAL OF PHILOSOPHY is owned by The Journal of Philosophy, Inc., 515 West 116th Street, New York, and published at Lancaster, Pa.: President, Wendell T. Bush; Vice-President, Herbert W. Schneider; Treasurer, Frederick J. E. Woodbridge; Secretary, Iris W. Wilder.

Entered as second class matter, March 18, 1921, at the Post Office at Lancaster, Pa., under Act of Congress of March 3, 1879.

"SCIENTIA"

INTERNATIONAL REVIEW OF SCIENTIFIC SYNTHESIS
Published every month (each number containing 100 to 120 pages)
Editor: **EUGENIO RIGNANO**

IS THE ONLY REVIEW the contributors to which are really international.

IS THE ONLY REVIEW that has a really world-wide circulation.

IS THE ONLY REVIEW of scientific synthesis and unification that deals with the fundamental questions of all sciences: the history of sciences, mathematics, astronomy, geology, physics, chemistry, biology, psychology and sociology.

IS THE ONLY REVIEW that by means of enquiries among the most eminent scientists and authors of all countries (*on the philosophical principles of the various sciences; On the most fundamental astronomical and physical questions of current interest, and in particular on relativity; On the contribution that the different countries have given to the development of various branches of knowledge; On the more important biological questions, and in particular on vitalism; On the social question; On the great international questions raised by the World War*), studies all the main problems discussed in intellectual circles all over the world, and represents at the same time the first attempt at an international organisation of philosophical and scientific progress.

IS THE ONLY REVIEW that among its contributors can boast of the most illustrious men of science in the whole world. A list of more than 350 of these is given in each number.

The articles are published in the language of the authors, and every number has *a supplement containing the French translation of all the articles that are not French.* The review is thus completely accessible to those who know only French. (*Write for a gratis specimen number* to the General Secretary of "Scientia," Milan, sending 1 sh. in stamps of your country merely to cover expenses.)

SUBSCRIPTION: **sh. 40; $10,** post free. OFFICE: Via A. Bertani, 14—Milan (26).
General Secretary: DR. PAOLO BONETTI.

Publishers: WILLIAMS & NORGATE, London—WILLIAMS & WILKINS CO., Baltimore—FELIX ALCAN, Paris—NICOLA ZANICHELLI, Bologna—RUIZ HERMANOS, Madrid—RENASCENCA PORTUGUESA, Porto—THE MARUZEN CO., Tokyo.

Vol. XXII., No. 17 August 13, 1925

THE JOURNAL OF PHILOSOPHY

MATHEMATICS AND CREDULITY

IN a pathetic outburst in the Introduction to his fallacious "New Theory of Parallels," the creator of matter-of-fact Alice and the elusive Snark exclaims: "The charm (of pure mathematics) lies chiefly . . . in the absolute *certainty* of its results: for that is what, beyond all mental treasures, the human intellect craves for. Let us only be sure of *something!* More light, more light! . . . 'And if our fate be death, give light and let us die!' This is the cry that, through all the ages, is going up from perplexed Humanity, and Science has little else to offer, that will really meet the demands of its votaries, than the conclusions of Pure Mathematics."[1]

The journeyman mathematician may not, in his sober interludes, feel certain of the coy snark concealed in twice two, but the ever-buoyant popularizer of the "inhuman science" has no such qualms. He finds the object of his search everywhere, whether it is there or not. To him the reputed "Queen of the Sciences" appears to rule her unwieldy domain with a rod of steel and a head of cast iron, and to be withal generous. When the enthusiast's moth-eaten creed rots and leaves him naked, the gracious Queen presents him with another, more durable than the first, for half of it is cotton and unpalatable to maggots.

"The genuine spirit of Mathesis," says Thomas Hill, speaking seriously and with obvious fervor, "is devout. No intellectual pursuit more truly leads to profound impressions of the existence of a Creator, and to a deep sense of our filial relations to him, than the study of these abstract sciences." And John Arbuthnot zealously declares that "Mathematical knowledge adds vigour to the mind, frees it from prejudice, credulity, and superstition." Viewed from the side, these remarkable utterances, like the legs of a decapitated pylon, appear to be mutually supporting. A simple transformation of coördinates—a slight change in the point of view, however, lends credibility to the suspicion that they are as outrageously contradictory as the excluded middle. Again, we are asked by Edward Everett—not a mathematician, by the way—to believe that "In pure

[1] This, and the quotations from Hill, Arbuthnot, Schellbach, Everett, and Pringsheim are taken from the *Memorabilia Mathematica* of R. E. Moritz, 1914.

mathematics we contemplate absolute truths, which existed in the divine mind before the morning stars sang together, and which will continue to exist there, when the last of their radiant host shall have fallen from heaven''; and C. H. Schellbach testifies like any bigot that ''He who knows not mathematics and the result of recent scientific research dies without knowing *truth.*'' Did opinionated assertion silence Pilate?

''Let us only be sure of *something!*'' Is not this the thick cry of a disgruntled Anglican smothering under the soft fatuities of a too protracted Oxford movement? Nevertheless, if a mysterious something or other is indispensable to the intellectual peace of the tender-minded, Mathematics can furnish the necessary soothing syrup. Alfred Pringsheim, a professional mathematician, offers us the *cliché:* ''It is true that mathematics, owing to the fact that its whole content is built up by means of purely logical deduction from a small number of universally comprehended principles, has not unjustly been designated as the science of the *self-evident.*'' If to be true is formally equivalent to being self-evident, then we have found the inestimable ''something,'' and Pilate is effectively answered.

Is the ''axiom of infinity'' to be received into Pringsheim's select society of ''universally comprehended principles''? If it is, then the claims of Arbuthnot, Hill, Everett, Schellbach and the uncounted host of popularizers of mathematics who have mistaken equivocations for equations, may be more than sterile flowers of rhetoric. But should the ''axiom of infinity'' be so fatally self-evident that it is no more than a verbal proposition on a par with ''quadruped = four-legged beast,'' then it would seem that we must apply to mathematics, as to physical speculation, the less exuberant estimate of Eddington:[2] ''We have found a strange footprint on the shores of the unknown. We have devised profound theories, one after another, to account for its origin. At last we have succeeded in reconstructing the creature that made the foot-print. And lo! it is our own.'' If, indeed, this mysterious footprint is our own, then Thomas Hill's ''deep sense of our filial relations to him'' becomes a singularly intimate form of ancestor worship, bordering on conceit; the Andromeda nebula shall be outlasted, not by the multiplication table existing from everlasting to everlasting like a disembodied chessboard in the divine mind, but by Edward Everett's ability to see stars; Dodgson's ''absolute *certainty*'' trails off into but another instance of the lovable human trait of glorifying *meum* at the expense of *tuum*—the mathematician's ''mine'' bragged up against the non-mathematician's ''thine,'' and ''the cry that, through all the

[2] *Space, Time and Gravitation,* p. 201.

ages, is going up from perplexed Humanity'' is solaced with a fraud, for all the encomiums of pure mathematics by its professional *claqueurs* are no better than preposterous bosh. Grant that the ''axiom of infinity'' is precisely what its name implies, a pure assumption, and you admit that the ''certainty'' of a major part of mathematics, if not all of mathematical analysis, is no greater than that of any physical science. And more than one reputable mathematician, as will appear in the sequel, admits as much.

It is no new contention that the ''generality'' or ''universality'' of mathematical conclusions is greatly exaggerated, not only in the minds of laymen, but also in the writings of those mathematical philosophers—or philosophical mathematicians, whichever may be to them the less opprobrious epithet—who exalt mathematics at the expense of other sciences with which, in respect of validity, it is on no higher than an equal footing. As the reiteration of such grandiose claims continues to be a favorite and profitable pastime for the popularizers of mathematics, and as it is, moreover, a constant irritation to many working mathematicians who are blessed with a sane skepticism—who not only see their science, but who also occasionally see through it—I believe it will be doing mathematics more than a slack lip service to outline briefly one part of the evidence which renders nugatory the claim that the conclusions of mathematics are a less legitimate object for skepticism than are those of political, physical, or religious speculation. Incidentally the little nosegay of quotations presented here may incite some logician with a sense of the ridiculous to cull his own anthology and, with it as an inspiration, to prick the wind bladders of the over-enthusiastic.

Mathematical induction, or reasoning by recurrence, is characterized by Poincaré as ''mathematical reasoning par excellence.''[3] The process is as follows: (1) We first establish a theorem for $n=1$. (2) Then we show that if it is true of $n-1$ it is true of n. (3) From (1) and (2) we conclude that the theorem is true for all values of n. The foregoing reasoning was Poincaré's; but he later, as we shall see presently, recanted on the shameless form of (3) as stated above. It seems to be admitted that before we can infer (3) from

[3] There is a discussion of this subject by the Editor and others, in the *American Mathematical Monthly*, Vol. 27 (1920), pp. 413–415; Vol. 29 (1922), pp. 162–164, where detailed references to Poincaré's well-known writings on this topic will be found. While preparing the note which started the discussion, I was perfectly familiar with the Literature cited by the others who took part, and with the theory of the ancestral relation in *Principia Mathematica*. A careful reconsideration of all the points mentioned has yielded no grounds for a revision of the position stated in the first note, which is, moreover, a commonplace to at least one school of mathematical philosophers, those whom Borel designates as the ''finitists.''

(1) and (2) we must state explicitly, as a postulate or as a definition, the following or its equivalent. (4) If a theorem is true for $n=1$, and if its truth for $n-1$ implies that it is true for n, then the theorem is true for all whole numbers. This amounts, of course, to a *definition* of "all whole numbers." But, like any other definition, it is no guarantee against future fallacies. Nor is it necessarily an act of creation. Any honest woman who has enjoyed the experience will tell you that having a baby is quite a different thing from defining one. With (1), (2) alone we can not step from $n=10$ to $n=12$ without the intermediate $n=11$; to attempt to do so puts the conclusion for $n=12$ into the category of things hoped for, the domain of credulity.

A fuller statement of Poincaré's position is contained in the following quotation:

"We may readily pass from one enunciation to another, and thus give ourselves the illusion of having proved that reasoning by recurrence is legitimate. But we shall always be brought to a full stop—we shall come to an indemonstrable axiom, which will at bottom be but the proposition we have had to prove translated into another language."

It is sometimes replied to this that what we are talking about when we say "all whole numbers" is merely what we have agreed to name "all whole numbers," that is, the class of things which have inductive properties as defined in "reasoning by recurrence." The existence of "all whole numbers" beyond a known number, say 10, is irrelevant to the truth-value of our propositions about the *defined* numbers beyond 10. In that case our propositions give us little or no information we did not already possess before we proved them, except possibly tautologies about a hypothetical null class. The majority of mathematicians, however, seem to consider this way out as a mere quibble, and hold that "all whole numbers" is neither one of Russell's famous "meaningless noises," nor irrelevant to the proposition proved as regards existence. A further dispute on the meaning of "existence" in this connection, offering another possible reply, would take us too far afield here, where our purpose is merely to indicate the interpretation put upon reasoning by recurrence, and upon the limits of its "certainty," by mathematicians.

With Poincaré's statement we may compare the following remarks of Russell:[4]

"The use of mathematical induction in demonstration was, in the past, something of a mystery. There seemed no reasonable doubt

[4] *Introduction to Mathematical Philosophy*, p. 14. This popular statement of his position is sufficiently detailed for our present purpose.

that it was a valid method of proof, but no one quite knew why it was valid. Some believed it to be really a case of induction, in the sense in which that word is used in logic. Poincaré considered it to be a principle of the utmost importance, by means of which an infinite number of syllogisms could be condensed into one argument. We now know that all such views are mistaken, and that mathematical induction is a definition, not a principle. . . . We *define* the 'natural numbers' as those to which proofs by mathematical induction can be applied, *i.e.*, as those that possess inductive properties. It follows that such proofs can be applied to the natural numbers, not in virtue of any mysterious intuition or axiom or principle, but as a purely verbal proposition."

The reader may decide for himself whether the animadversion in the foregoing quotation is a just statement of Poincaré's views as stated above. For later reference we cite Russell's next sentence.

"If 'quadrupeds' are defined as animals having four legs, it will follow that animals that have four legs are quadrupeds; and the case of numbers that obey mathematical induction is exactly similar."

A few lines later he strikes at the root of the matter, a root which remained hidden till Cantor, Dedekind and Weierstrass dug it up and labeled it—another of these potent "definitions"—when he says that "mathematical induction affords more than anything else, the essential characteristic by which the finite is distinguished from the infinite."

Nothing could be clearer. Before we can induce the obstinate layman to swallow the whole of a proof by mathematical induction we must overcome his merely finite objections by choking him into coma with the axiom of infinity. We shall return presently to this interesting operation.

In the meantime let us note that Pringsheim's "small number of universally comprehended principles" is beginning to flirt with the universally incomprehensible. Even the expert mathematician confesses to a possible haziness of perception regarding the newcomer. Thus Borel,[5] remarks that ". . . in spite of the difficulties presented by the series of *all* whole numbers, in spite of the material impossibility of ever passing beyond a finite rank in this series, we can imagine, more or less clearly, its indefinite prolongation." When we begin imagining things "more or less clearly," we may as well admit that we are seeing them as through a glass darkly. Having opened the bag, Borel, like a good mathematician, incontinently lets out the cat, as the only logical conclusion. "Whenever, on the contrary," he continues, "it is a question of a non-denumerable infinity, I must con-

[5] *Leçons sur la théorie des fonctions*, Paris, 1914, p. 179.

fess that all the verbiage (*discours*) by which they have tried to arouse in me the idea of it has never suggested to my imagination anything but the enumerable infinity; reasonings on the aleph symbols therefore maintain for me a purely abstract character, corresponding to no reality whatever." This, and similar confessions which the reader who is interested can collect for himself, indicate that mathematics " as the science of the *self-evident*," in Pringsheim's phrase, is an outworn tradition. They suggest also that not only is the sequence of "all whole numbers" in a haze, but also a considerable part of mathematical analysis. It also, beyond the rudiments, is a legitimate object for rational skepticism, like any other of the innumerable answers to "the cry that, through all the ages, is going up from perplexed humanity." For one school, at least, of mathematical analysts feels confidence in its conclusions only when the entire chain of deductions necessary to reach them has been "arithmetized" in accordance with Kronecker's classic program,[6] which would restate every theorem in analysis as a relation between whole numbers only. To reach " general " theorems of analysis by this road we must again lean on "all whole numbers," and, it would seem (for example in discussing "functional space"), on the more mystical transfinite. (I have used "mystical" in its religious significance, as a transcendental intuition of the inexperienceable.) That Kronecker's genial revolution has been characterized somewhat peevishly[7] as "Mathematical Nihilism," but adds to its spiritual charm. It is, of course, no disgrace to any part of mathematics that it should be an object for credulity or its opposite, for whatever is worth thinking is worth doubting, and a thoroughgoing doubt is one of the highest compliments that can be paid to any thought; it shows, at least, that the thought is not merely pretty. But it is otherwise with faith masquerading as fact.

Having admitted one infinity into our science of the self-evident we may resign ourselves to welcoming the whole brood. The ten years' war between Poincaré and the logisticians ended, and remains so ended, with fashion favoring the latter. Like the above quotations from Russell and Poincaré, this famous debate gave a pragmatic demonstration of one thing; experts disagree as to what meaning is to be assigned that subtle phrase "a meaningless noise." The polemics, ably summarized by Borel, over the whole swarm of infinities let into mathematics by incautiously opening the door to mathematical induction, but emphasize the disagreement, and suggest a revision of Lord Kelvin's optimistic estimate that "mathematics is

[6] "Ueber den Zahlenbegriff," *Journal für Mathematik*, Vol. 101 (1887), pp. 337–355.

[7] By E. W. Hobson, *Theory of Functions of a Real Variable*, 1st ed., p. 22.

the only true metaphysics." Kelvin was a mathematical physicist rather than a pure mathematician; to a pure mathematician all metaphysics, even his own, are impure. The question of mathematical induction has dribbled off into epistemology.[8]

Let us now return to Russell's quadrupeds, and *define* pentapods to be five-legged beasts, taking as our fundamental postulate the nonexistence of such. In this we are only slightly less disingenuous than the mathematician who states candidly, as does Russell (*loc. cit.*, p. 75), that the *existence* of infinite collections is a pure assumption. It follows that the class of pentapods is the null class of beasts. In a universe of bestial discourse our propositions about pentapods degenerate to tautologies concerning the null class. Instead of quadrupeds or inductive numbers we may consider round squares, whose existence any competent metaphysician can demonstrate with ease and without appeal to the cardinal article of mathematical *faith*, the existence of infinite collections. The best, apparently, that can be said for their existence seems to be Russell's "there is no logical reason *against* infinite collections," which is a spineless kind of existence. Nor was there any logical reason against the inverse square law of gravitation. I have juxtaposed these examples and quotations to foreshadow the latent mares' nests in the following substitute for Russell's or Poincaré's brands of "mathematical reasoning par excellence," and to emphasize the necessity for *belief* (nothing stronger) in the unprovable (Russell, *loc. cit.*, p. 77) axiom of infinity in any discussion of mathematical induction which aims at more

[8] In passing we may note a delightful instance of table turning in connection with Lord Kelvin's contempt for metaphysics. It must be a joy to any good Hegelian to overhear the continuation in the Elysian Fields of Kelvin's rather one-sided controversy with Hegel. This "arrant imposter" Hegel (so characterized in the *Life of Lord Kelvin*, Vol. 2, p. 1122, by the mathematical physicist) seems to have the last laugh. For Kelvin could scarcely have selected a more unfortunate passage by which to hold Hegel's physics up to ridicule than that which he chose, Hegel's famous attack on the Newtonian theory of gravitation: "*The motion of the heavenly bodies is not a being pulled this way, or that (such as imagined by the Newtonians), but is free motion; they go along as the ancients said, as blessed gods.*" There is more also in the rest of the quotation which, when we substitute for Hegel's metaphor, "*celestial corporeity*" its modern equivalent, an everywhere (except at an infinite distance from all material bodies, or Hegel's blessed gods) curved four-dimensional continuous manifold of timespace, becomes as good a popularization of Einstein's theory of gravitation as any that has yet appeared. The planets are *not* being mauled about by Newtonian "forces," the relativists believe, but *are*, as Hegel declares, moving freely along geodesics in timespace. "Hear his words!" Hegel may exclaim to the celestial umpires, taking the exclamation from his traducer's lips (*loc. cit.*, p. 1123). "If, gentlemen, these be his physics, think what his metaphysics must be!"

than disguised nonsense. The following is from Hobson, (*loc. cit.*, p. 7, repeated in the second edition):

"If, in respect of a given simply ordered aggregate it be known (1) that in case any element possesses a certain property P, the element of the next higher rank possesses the property P, (2) that the element of lowest rank possesses the property P; it then follows that every element of the aggregate possesses the property P."

This is proved easily by a simple contradiction from the *definition:*

"A simply infinite ascending aggregate, or simple sequence, is an ordered aggregate which has no element of higher rank than all the others, and is such that every part which has an element of higher rank than all the other elements in that part is a finite ordered aggregate."

Is not "all" in the phrases "all the others," and "all the other elements," used in different meanings, one referring to the number of an infinite aggregate, the other to that of a finite aggregate? Granting the existence of the first we may presume that that of the second is intended to be a definition or an axiom within a definition. It is, in fact, our familiar friend, the axiom of infinity. And do we not further assume that such an aggregate and its pair of "all's" can coexist like an unholy trinity without possibility of concealed contradictions of type, as in the statement by a certain Cretan that "all" Cretans are liars? Assuming this much, namely, that the class of infinite ascending aggregates is not the null class of aggregates, we have assumed what we wished to prove, and either our propositions, deduced by means of the substitute principle (a consequence of the definition), are tautologies about the null class, or we must make a new definition, or further assumption, that the class of pentapods or inductive members, as the case may be, is not null. This brings us back to Poincaré's "full stop"—the proposition to be proved restated in other words, and to Russell's definition of inductive numbers. As before, we find the object which our demonstration would fix eluding us and receding farther and farther into the dim regions of faith. Incidentally we have been brought to a realization that it is not merely in elementary texts and popular spasms that an underhand attempt is made to foist upon the naturally skeptic mind a plausible sham for a repugnant credulity.

Whether Poincaré or Russell is right, mathematical reasoning probably is not the supernatural engine which the layman and his prophet, the popularizer, think it is; it fails completely to justify their superstitious reverence for mathematical rigor and the "absolute *certainty*" of pure mathematics. The one thing of which

we are certain seems to be an uncertainty as to everything except this itself. Most mathematicians, of course, are fully cognizant of the gibe that the mathematical machine can grind out only what they (not the Creator) put into it, and are, as a rule, under no illusions as to the grist from their mill. But there are exceptions, and it is these whom many a misguided layman or tyro in mathematics mistakes as his guides and philosophers. To justify the belief of these misguided beings we must present them with a Greek gift, the axiom of infinity; they must *assume* that the *all* furnished by induction exists; they are entitled to a baseless *faith*, no more, in "what, beyond all mental treasures, the human intellect craves for." The man who sees clearly with Poincaré that in proving a mathematical proposition inductively he has merely translated the hazy into the hazier, or the one who realizes with Russell that he has been elaborately disguising a definition, is not likely, unless his ears be inordinately long, to hear in the "absolute certainties" of mathematics anything more reassuring or less meretricious than those reached by precisely similar means from unprovable hypotheses in the electronic reactions of Abrams. That the truths of mathematics are no more reliable than the logic by which they are deduced from unproved assumptions is a trite truism, and although it seems to be the greatest possible to any science or to any creed, it is in itself no great virtue; the more ably conceived frauds of fact or of faith can justly claim as much. Wherein mathematics differs from other branches of science is the preposterous claim of universality and infallibility in its behalf put forth by the popularizers who would exalt it, not to the dubious station of Queen of Sciences, but to the baser eminence of Pope of Dogmas. Devotion to mathematics is said to lead to intellectual honesty. From some of the blossoms in our sample anthology a dispassionate observer might infer the reverse. It certainly seems to augment "the will to believe." Credulity is a fool's science.

Although mathematics with its infinite fails to satisfy the hyperrational aspirations of a rational mind (granting that such a mind can be capable of such nonsense), it is otherwise with the pragmatic test of applicability to the world as mathematicians perceive it. Admitting that *logically* the position of the finitists is very strong, Borel (*loc. cit.*, p. 177) justifies the use of the doubtful element as follows, and probably most students of mathematics will agree with him.

"It suffices to have studied the elements of mathematical analysis to form an estimate of the degree of complication to which the pretense of excluding the infinite, as it is considered in these elements, would lead."

Granted; but this does not justify a mathematical philosopher in deifying a common utensil, or in mistaking, as some of them do occasionally, a hod for a god. Superstition has been kicked out of the front door by the physical and biological sciences. Why should mathematicians permit it to re-enter the House of Solomon by the cellar window? Charity, perhaps, or possibly just plain sloth. So far as mathematics is concerned in these brawling days of peace, when Fundamentalists and Modernists are at each other's throats, it may be well to remember that no cause, scientific or spiritual, was ever helped for long by fraudulent claims in its behalf. That some mathematicians have stooped to make of mathematics a specious pleader in a cause where its voice is at best irrelevant, is a poor compliment to their science.

E. T. Bell.

University of Washington.

CLASSIFICATION AND DIVISION

IT is usually the notions with which we are most familiar that prove upon investigation to be the most inaccurately defined; we are convinced that we believe something, yet it is hard to say just *what* we believe. Classification and division have traditionally occupied a prominent place in logical treatises, and as a result have taken on a certain degree of familiarity. It is not at all unlikely that this very fact has led to their neglect from the point of view of rigid and concise definition. Logicians have been content to characterize them respectively as "synthesis" and "analysis," or "putting together" and "breaking up," and any further comment outside of an all too brief discussion of the meaning of such terms as *genus, species, sui generis, etc.*, was looked upon as elucidating the obvious. As a consequence the inquiring student finds himself at a loss to obtain material pertinent to a more detailed treatment of these notions. I propose in this paper to consider these terms from the point of view of the intensional definition of class, with a view to pointing out certain ambiguities and suggesting certain reforms.

To begin with classification, it seems clear that the word is ordinarily used in at least three senses. In the first place it is a name for the process by which an *object* or *particular*, in virtue of its possessing a specific attribute, is placed in the class defined by that attribute. A given particular, for example, which possesses the characteristic "redness" is associated with other particulars which are instances of the same quality, and the totality of such particulars constitutes extensionally the class "red." In this sense of the

word, all recognition is classification—unconscious classification if the recognition is immediate, and conscious if the recognition is mediated. All description, and all explanation, in so far as it is of a particular event, is simply a matter of classification; we identify an event by associating it with other events similar to it in certain respects. This is the commonest use of the word "classification," and I propose in what follows to employ it only in this sense. In the second place, classification is commonly used to designate the process by means of which a general or class term is placed within a term of greater scope. We say, for example, that men are "classed" as mortals, meaning by this that the class of men, together with certain other classes, constitutes the class of mortals. This use of the term involves only relationships of classes and never concerns individuals. Let us use the word "colligation" to designate this process. In the third place, the term is used synonymously with one or another of the three meanings of its correlative term, division. In this sense, for example, we frequently "classify" men into honest and dishonest.

The corresponding ambiguities in the term "division" are easily distinguished. Division means, in the first place, the process of specifying or enumerating the individual members of a class. This is the converse of the strict meaning of the term "classification." The outcome of this process is always particulars, which can be handled only by proper names or their equivalents in such words as "this" and "that." Let us call this "particularization." Corresponding to the second meaning of classification (what I have called colligation) division indicates the process of making explicit the several subgroups which lie implicit in any class. Formally this is done by "increasing" the intension of the term defining the class, as when the classes "tall men" and "short men" are distinguished within the class "men." This is the commonest meaning of division, and the word will be limited to this sense. The third use of the term has no analogue in classification. It refers to the process of physical dissection or partition which is involved in cutting up a whole, or attending to the parts rather than to the whole. This is possible in virtue of the fact to which Whitehead has called attention, that events "cover" or "extend over" other events. This will be designated by the term "analysis."

Now there is no question here as to the validity of these several meanings. Each notion is in some sense justifiable. But it is well to guard against confusing them one with another. In spite of their similarities, there are marked differences. The relation of individual to group is not the same as the relation of *species* to *genus*, nor is

either to be identified with the relation of part to whole. The tendency to confuse the first two is a direct result of the fact, much to be regretted, that traditional logic formally identified the universal proposition with the singular proposition. Since they could be identified with respect to the syllogism, it was thought that they need not be distinguished in any respect. The student of elementary logic was, as a result, compelled to struggle with the paradoxical statement that a proposition with a "particular" term as a subject was really "universal." The tendency to confuse the relation of *species* to *genus* with that of part to whole is easily explained by certain resemblances which they exhibit. The category of number applies to both. Just as the sum of the parts equals the whole, so the sum of the *species* (in a certain sense) equals the *genus*. Again, just as the whole is greater than any part, so the number of the *genus* must be greater than the number of any one *species* (provided the remaining *species* of this *genus* are not null). Since the category of number applies only to groups and not to individuals, the relation of individual to group can not be considered in this connection. There is a sense, however, in which a class can be said to be "made up" of individuals just as a whole is made up of parts.

It is the process of classification, strictly so-called, which gives rise to the notion of *sui generis*. An object is *sui generis* (the notion applies only to *objects*, and not to *classes*) when (*a*) it is the only member of its class, or (*b*) it is its own class. Three points may be suggested with regard to this notion. (1) The first of the defining characteristics enables us to readily identify a *sui generis* object as the one which is designated by the descriptive phrase "the so-and-so," provided there is such an object. The member of a unit class is always *sui generis*. (2) No object is absolutely *sui generis*, for every object must resemble some other object in at least one characteristic, and resemblance determines a class. Every *sui generis*, for example, lies in the class "existent objects" or "aspects of reality." (3) The second defining attribute of *sui generis* results in contradiction. If a class is identical with its members, then the following difficulty arises. The class of existing presidential cabinets has one member (*i.e.*, the present cabinet) and also ten members (*i.e.*, the Secretary of State, the Secretary of War, *etc.*) Hence a class is never to be identified with its members; the class is not the members and the members are not the class.

As a result of the intensional definition of class, division has broken away from classification and taken on new characteristics. According to the old notion one never divides a class into sub-classes without at the same time looking over the different sorts of objects

which might be put into these sub-classes, *i.e.*, classifying. Hence the number of *species* at any particular stage in the division is determined by the actual kinds of objects in existence. For example, I would divide tables in this room into the two classes, walnut and mahogany, because there are no tables of any other material here. But if we admit the intensional definition of class, and the resultant possibility of null classes, there is no reason for making division so dependent upon classification. The division is then carried out not on the basis of actuality, but simply with reference to possibility. My enumeration of classes in the example given would therefore include besides walnut and mahogany, also birch, oak, pine, and all other kinds of tables which might conceivably be in this room. All of these classes except those defined by the predicates, walnut and mahogany, would be null, but the classes themselves would not be non-existent.

As a result the notion of *infima species* reduces to an absurdity. For there is no point at which, according to the traditional notion, "it is impossible to carry on the division further without reaching the individual." This definition of *infima species* evidently involves in our terminology two points: (*a*) continued "breaking up" *can not* be division; (*b*) continued "breaking up" *must* be particularization. With regard to the former, since division, formally, is simply a process of specifying "determinates" within a certain "determinable" (to use Mr. Johnson's terms) there appears to be no reason in the nature of the *genus-species* division why characteristics may not be "added on" indefinitely. Metaphysically this means that no enumeration of universals will ever give the individual object; the object is more than the sum of its aspects. Hence, however many characteristics of a particular I enumerate in the description of it, I shall never "reach" the particular itself; a particular is never given by "description" alone, but is known only by "acquaintance." The result of a prolonged division may give either one of the following: (*a*) the class of which the individual proves (by reference to experience) to be the sole member, in which case it is well to remember that the individual is never identical with the class of which he is the sole member; or (*b*) the class of which (by reference to experience) there proves to be no members, in which case we must recall that the class is none the less real for its emptiness. Since both unit and null classes may be subdivided, there is no *infima species*. Furtherfore, particularization *may* be performed at any point in the process, but it is done only in virtue of having changed the plan of the "breaking up" from that of division to that of particularization.

A result which is even more surprising is the impossibility of

particularization apart from reference to experience. For it is only by reference to experience that one can determine whether a class has members. In the *genus-species* division, the null and the unit classes have the same status as the full classes. There is nothing in the definition of a class which tells us anything about its membership. As a further consequence, since deduction is simply a matter of relationship between classes, this form of reasoning can never give us a particular unless that particular is first inserted as a datum into the reasoning process. Deduction is not an instrument of discovery and can never give us a "new" particular. The well-known case of the discovery of the planet Neptune seems to be an exception; for here by means of mathematical laws and certain particulars, another particular was deduced. Actually, however, such was not the case; the deduction gave us not Neptune itself, but a class of objects defined in a certain way, of which Neptune proved by experience to be the sole member. But Neptune was not this class, and the deduction did not give Neptune. Thought can not determine existence. This seems to me to be one meaning of the statement that deduction can not give us new knowledge.

Analysis needs a word of comment because it is often confused with division and particularization. It is not unusual, for example, to find the analysis of the United States government into State governments, districts and precincts given as an instance of *genus-species* division. This is incorrect for it lacks one of the essential characteristics of *genus-species* division, viz., the possibility of designating each sub-class by the defining characteristics of all of the classes above. The State government is not the United States government. Furthermore, the United States government is not a class at all, but a particular; it could therefore be classified or analyzed, but never divided. An analogous misconception is involved in the notion that the class "electron" is an *infima species*. To refute this conception it is sufficient simply to suggest the classes "electrons in the earth" and "electrons in the sun" as sub-classes. It is true that "electron" can not be analyzed, but analysis is not division. It is true also that the class "electron" can not be particularized, for we have no means as yet of specifying a particular electron which we can indicate by "this," "that," or a proper name.

It may be contended that I am unnecessarily limiting the term "analysis," when I use it to designate the whole-part relationship. Is it not true that there are legitimate cases of the word which are not so narrowly confined? For example, it is possible to analyze the notion of happiness. Analysis in this connection means simply the consideration of the notion with a view to removing vagueness and ambiguity by dissociating from the concept unessential characteris-

tics, irrelevant associations, and misconceptions. In this sense, analysis is always preliminary to definition. Again, it is possible to analyze the notion of, say, sugar into its aspects, sweetness, brittleness, whiteness, *etc.*, yet these aspects are not part of the sugar in the same sense that the carbon and hydrogen atoms are. Analysis, therefore, should not be confined to the natural scientist's use of the term. But all of this discussion is, strictly speaking, irrelevant. I have tried, not to limit the term "analysis" to the physical whole-part relationship, but to exclude it from the *genus-species* and group-individual relationships. I have tried to show merely that the whole-part relationship, whatever it be called, should be sharply distinguished from certain other types of logical relationship.

The practical value of these distinctions is many-sided. To take the most outstanding example, reasoning according to the *dictum de omni* is clearly applicable, *in general,* only to the *genus-species* relation. It can not be stated as universally true that "what is true of a class is also true of a part of the class." Logicians have formulated this principle, and then, realizing its inadequacy, have felt called upon to point out two types of fallacy inherent in it. These were called, respectively, composition and division, and were shown to be based upon the ambiguity of the word "all." Fallacies of this type illustrate the two important facts that many qualities which can be predicated of classes can not be predicated of the individual members of the classes, and that many of the qualities of a whole can not be attributed to a part of that whole. An example of the former, taken from Russell, is the following: Since the Greeks are numerous, and Socrates is a Greek, therefore Socrates is numerous. Number is a characteristic of classes only, and an individual *qua* individual does not have number. On the other hand, the category of existence is applicable only to individuals, and can be attributed to classes only by first designating the sense in which it is to be used. The fallacy of attributing to a part the characteristics of the whole is seen in the following: Since the team is strong and John, James . . . are the team, therefore John, James . . . are strong. The parts in disunion do not have the same characteristics as the parts in union.

Ultimately, of course, the confusion between the three types of relation is due to the ambiguity of verb "to be." It is a most unfortunate fact of our language that one verb should be used to designate three such distinct relations as "Socrates is a Greek," "The Greeks are numerous," and "John, James . . . are the team." It is one of the outstanding contributions of symbolic logic to have pointed out and turned attention to these confusions.

A. Cornelius Benjamin.

University of Illinois.

BOOK REVIEWS

University of California Publications in Philosophy; Vol. IV (1923); Vol. V (1924). George P. Adams, J. Loewenberg, editors. Berkeley: University of California Press. 224 pp. 220 pp.

Anatole France had various ways of saying, the one period beyond history's reach is the present. And yet some of us are so given to making the past out of which a day has risen part of that day's meaning, we can not let an expression of even the most present present pass without venturing a speculation on how the future will look back on it.

Of the group of writings before us, this posterity must have at least one grateful comment to record; it must remark the ease with which any decently trained mind can follow the discussion for what it is worth without having first to master an idiom-of-the-cenacle. If this later day shall remain as conscientious in its demands on the interpreter of historic moments as (we hope) our own has been, for sure there will be interpreters in it who will find the first quarter of the twentieth century more "available" for seminar purposes than the like quarter of the nineteenth.

But perhaps in another point of contrast these descendants of ours will miss something whose loss they will not count to our profit: if they are at all heir to the older men of this day they may feel as we do the poverty of historic reference in so much of the writing of its last moment. I wonder how they will account for the prodigality of current allusion our writers allow themselves at the expense of all large orientation, all drama of thought's evolution—in such contrast with the patient effort of certain other epochs, to show why ideas *had* to develop toward a certain fruition. But very likely these future historians will content themselves with a simple "reaction-motive": from having had too much Hegel, they will say, the early twentieth came to have too little history; with a like impious joy did the seventeenth century emancipate itself from the historicality of the Renaissance. Though let us hope that day will have recovered sufficient control over the ironic use of historic material to put behind its ideas something of that developmental push a Plato or an Aristotle found so to his hand, in the days before Hegel had improved things to death. However that may prove, the reviewer confesses to a certain sense of reassurance when he comes upon a writer who, like Loewenberg, can suspend his contemporaneity long enough to catch certain points of likeness between what is being mooted round about and what has been worked over with some pains several times in the past. Then, of course, Lenzen does all that could be asked of

one presenting new conceptions of physics, in showing out of what past images well-seasoned motives of reflection have advanced the scientific thinker to a closer realization of old ideals. It is curious how scientific exposition has come to care so much for the "push of the past" at the very moment technical philosophy has ceased to derive impetus from its own historic motives.

And this makes it all the more curious that our current writers are in large number only too eager to accept at hands they take to be those of natural philosophy an idea quite foreign to the older spirit whether of science or of philosophy. Nothing is further from the sympathy of this group of writers than what thought or what nature "non faciat saltum." Here knowledge jumps to conclusions with a new gaiety of heart and nature leaps chasms with a spritely irresponsibility. We may imagine our future historian, if he let a word's frequency measure its importance, to characterize the thought of the day as a "philosophy of discontinuities," whose proponents he may like to call, the "chasmists." It may be well, then, for continuity's sake to center our reflections on these discontinuities; and, first, of epistemological breaks.

There has always been a certain difficulty, whose expression changed with the times, in inducing the idea of a thing and the thing of which this was to be the idea, to share a common world. And yet if both really were, there should be some world of reality in which both were at home. Lovejoy misses nothing of what might make this ancient difficulty striking when for the *idea* he selects a present memory-image, whose vanished object becomes the *thing*. There is here a double "anomaly":

> (1) The lesser arises when you view the phenomenon of knowing from the standpoint of physical science; and it consists in the fact that there exists an organism, itself physical, which at some moments of its life has as factors in its environment things that are not, at those moments, any part of the physical world.
>
> (2) The greater anomaly consists in that peculiarity of knowing, which philosophers call 'meaning' or 'transcendent reference'; that is, in the fact that when we know we appear somehow to have within the field of our experience objects which we must at the same time conceive as having their existence entirely outside that field—for example, as having their being at a time other than the time of the knowing of them (IV; 15; condensed).

As preparing the way for a solution of the major anomaly, the author devotes himself to a psychology of the memory-image: how can a mental state all present at once so much as *pretend* to represent pastness? Were it simple and unanalyzable, doubtless it could not, but

> the content given in memory is not a simple thing; elements presented simul-

taneously are presented *as* not simultaneous. The present images have two dates —their date of existence, which is today, and their date of reference, which is yesterday. But the pastness of their date of reference is itself a present quality of the memory-images. It is something which I directly experience as one of the attributes of the now given content. . . .

(Let me interrupt to observe that so far we have only a *Fragestellung*, one with which no classic defender of "representative ideas" would be likely to find fault; he would merely wait anxiously to learn *what* "present quality" the image could have that would serve to give a "date of reference"; and he is answered. . . .)

We apprehend the various elements of our presented content as fitting into a framework of *conceptualized* temporal relations—which in fact appear in consciousness largely in the form of spatial imagery, as of a calendar or a timetable. [And our author recalls Royce's 'self-representative thought' which, like the map of a city placed within the city mapped, shows itself in its own midst. Just so. . . .] A given moment of thought may consist in the representation of a whole world of objects in which it is itself, *as represented*, a mere fragment (39; condensed).

If I have brought forward this psychology of a certain "representative idea" to introduce a display of epistemological jumps, it is not that the author ventures the jump on the occasion we are reviewing. "The question [he explains] how tests of *validity* in our knowledge of not-present objects can be applied . . . lies beyond the scope of this lecture" (42N.). But it is hard to see how Lovejoy should proceed to this "test of validity" in a way different from any suggested by his fellow-contributors to the volume, *Essays in Critical Realism*, in commenting on which Loewenberg is led to cite this "inter-temporal cognition, the reference of one moment's experience to that of another moment" (Lovejoy, *Essays*, 53) as a typical instance of what is involved in knowledge as such, as seen by critical realists: . . . "Most of them actually make their appeal to such inter-temporal cognition to illustrate the difference so crucial for their doctrine between 'object' and 'content'" (IV; 170).

With whatever jumpiness one may await this "test of validity," the critical realist's transition from content to object can not disappoint it. It is true, "not all of the critical realists have Santayana's engaging candor in speaking of belief [in the idea's object] as a 'leap'" (172). ("Curious, Loewenberg comments, that such leaps, however coercive, whether of 'intuition' or of 'action,' should be considered a satisfactory ground for a belief in the existence of an external world.") But the alternative espoused by other of the critical realists is no less "curious"—it involves and "makes necessary the introduction of those mysterious 'essences,' the *dei ex machina*, without whose aid neither true nor false cognition could be accounted for."

> These essences parading under various synonyms, such as qualities, character-complexes, character-traits, logical entities, logical universals, and being thus radically different from 'copies' of the older representative theories, are indispensable as links between independent outer objects and their counterparts in cognition (171). . . .
>
> [The critical realist] wants to confine knowledge to characters capable of being held before the mind (Rogers, *Essays*, 157), and he wants also an external world, independent of the mind in which these essences may or may not have an existential habitat. The logic of the situation forces him to seek in the essences, which are the only data the mind has, transcendence as a necessary ingredient (173).

Perhaps there are those who would think the *link* even "curioser" than the *leap*, did it not come to us with a certain warmth and intimacy. For this, our sense of "having been here before," Loewenberg (who as we said *uses* history) has no difficulty in accounting:

> [This urging of essence to establish existence] is a case of the familiar attempt repeatedly made in the history of philosophy, to compel the 'what' to yield a 'that.' In this particular use of transcendence by Critical Realism we seem to have *a faint suggestion of the famous ontological argument* (173, italics mine).

"Only," this historically-minded critic concludes, "it is not argument. It is ontological faith." The reviewer might wish to put the matter a little differently: *Either* the "link," *or* the "leap"! Either this is *argument*, in which case it is not "faintly" but flagrantly suggestive of rationalistic attempts "repeatedly made in the history of philosophy." Or else it is *faith*—a faith suggestive of irrationalities repeatedly invoked in the history of human difficulties.

This "leap" then, and not the "link," would seem to be the novelty offered by the critical realists. "Novelty," not because such transcendence of evidence is new to man's history; but because it has generally been supposed to belong to that part of man's history of which philosophy has least reason to feel proud. (I think no one would point to the *intuitus* of Descartes as forebear of the *leap*, whatever its responsibility toward the *link*). Whether or not the "queerness of knowledge, its distinctiveness among natural processes" (IV; 43), which necessitates such leaps of epistemology, also necessitates or merely encourages that general "jumpiness" the school finds in nature, it is certain this leaping epistemology lands in an objective world replete with "chasms."

Adams, proposing "to set forth certain considerations in the light of which the entire life and habitat of mind are not adequately to be described in terms of the intercourse between animal bodies and their physical environment," enumerates the following (condensed):

> 1. Where a stimulus leads uninterruptedly to its appropriate response, as in the case of instinctive behavior, the continuity between the organism and its world is maintained. The origin of consciousness would appear to be coinci-

dent with some break in the continuity between organism and environment. Instead of a physiological mechanism interacting with objects immediately and directly we have a self holding commerce with its world *through the medium of ideas.* Between the life processes of the organism and the objects which impinge upon them, there comes to be inserted the region of ideas, of contents of consciousness (IV; 56, italics mine).

2. If reality is envisaged solely in terms of bodily life processes interacting with a physical environment, there is not only no past in the sense in which there is always a moving present, but what once existed and is now real as a past event could not be known to have been so. The distant in time cannot itself impinge upon any present process (60).

3. A second instance of a dimension which cannot be disclosed to any physical process, and which is revealed through ideas, is that of universals. . . . While the particular thing of process, including the configuration [*i.e.*, characterization in universal terms] impinge upon the body in the space-time continuum, cannot see that the configuration, the universal as such, does so. Nature let fall the apple on Newton's head, but not the law of gravitation (61).

4. In addition to the real past and universals, there is a third class of entities which I find difficulty in locating in any process within the space-time continuum. I refer to meanings. The *apprehension of meaning* is a process. But the *meaning as apprehended* is a structure which possesses a fixity, an immobility, which renders it unfit to be a moment of or a cross-section of a process (62, italics mine).

The last argument serves as a sort of transition to "a further consideration which bears upon the adequacy of the organizing concept of bodily processes continuous with nature's processes" (64)—and back of this lies Adams' own philosophy of "nature and the habitat of the mind." Let me reserve this for special remark, for it seems to me of a different order of dignity, and of other and richer suggestion. Indeed, were it not for this "further consideration" the argument would invite no special reconsideration. No one, it is safe to say, who has been led to see nature's development—life, mind, consciousness—as consistent with one unbroken tale of mechanism, can have been misled into doing so by having let pass unnoticed any of the points here numbered with such care. That instinctive reactions to environment are in some sense less "immediate," less right-off-the-bat than "intelligent," "thoughtful," "conscious" copings with situations, is as good a reason as another for our calling them "instinctive." That the past is in some sense no longer here to act on the organism of the moment, is why we refer to it as "passed." That universals ne sautent pas aux yeux, that a law of gravitation does not fall like an apple on the head, is an historic motive for inventing "universals." That meanings mean to lay hold on the eternal, is part of their good intention; if they could they would grasp the *Ding an sich* and embrace *die Sache selbst;* but our finite minds can only mean well by these ideals.

If, then, there are those to whom these obvious considerations

bearing on mind and its functioning suggest no such "breaks or 'chasms' [as would let nature's history] exhibit the 'emergence' of absolute novelties, discontinuous variations in no way deducible from, or explicable by, any characters of the prior members of the series" (Lovejoy, V; 176)—the reason for such a difference of philosophies ought itself to be a philosophy, and an interesting one.[1]

For those who have but two boxes into which to put existences, the one receiving *bodies* (with such attributes as position, mass, charge, velocity); the other, *minds*, (with their "strange activities" and "queernesses of knowing" (IV; 43)) there is nothing to do but dwell on the discontinuity, break, chasm, abyss, yawning between mind and body: this can not be portrayed deeper than it is. But to one to whom "object" has long ceased to suggest "body" (as above defined) rather than certain definable functions of the observable behavior of groups and classes-of-groups of such "bodies"—minds may well belong to the *same objective order as bodies;* their appearance in nature may well consist with a smooth story of mechanism. If one ask, why do these philosophies remain so unreconciled?, the answer would seem to be that, whereas neither the "objectivist" nor any one else could have overlooked the obvious and historic considerations the "chasmist" dwells on, the "chasmist" seems quite unwilling to think out the meaning of the alternative. Consequently, his criticisms are not so much misdirected as wanting direction. Suppose, for example, he has confined his critical attention to a "best known representative of behaviorism, [who], seeking some particular motion of matter as the fact corresponding to what has commonly been called 'knowing,' finds it in the movements of [certain] muscular mechanisms." Suppose, then, the critic to interpret the behaviorist in question as believing that "when you have described these [mechanical movements] you have told the whole story about thought and knowledge" (IV; 26, condensed). What should a thinker conclude, on this data, as to the soundness of such behaviorism? Exactly nothing—nothing favorable, nothing adverse! The account fails to furnish judgment with the one bit of information on which its decision could be formed. It has not told us *how*, in what categories dependent on what relationships, the "motions" in question were "described." It seems inconceivable that a logician should have failed to observe to what extent the description makes the thing—yet the "objectivist" has so far suffered most from the lack of criticism

[1] For a fuller discussion inspired by this topic ("Discontinuities of Evolution," V; 173 ff.), the reviewer ventures to refer to his paper "Logic, and the Relation of Life to Mechanism," *Proc. Am. Phil. Society*, LXIV, No. 1 (in press).

meaningful enough to allow him to recognize in its object, I do not say his own, but *any* doctrine.

At the risk of revealing an amiable naïveté, let me smooth matters over with this self-solacing remark: where the "objectivist" *does* recognize something like himself, he does not find his kind credited with absurdities. There is a way of looking on nature attributed by Kemp Smith to Whitehead, whose "philosophy of nature" is passing in review (IV; 197 ff.).

> The organism must, [Whitehead] maintains, be treated in the same general manner as atoms and molecules. By its existence it demonstrates the physical reality of certain rhythmic functionings which, as such, are incapable of being enclosed within the instantaneous. As rhythmic, they presuppose duration as truly as does a melody. 'Life is too obstinately concrete' to be located in an instant, even if at the instant it be spread over space. For this reason, living organisms, for their definition, demand synoptic as well as analytic treatment. In so far as they are complex, analysis is required to determine the precise nature of their complexities; but in so far as they are rhythmic, these complexities are subordinate to, and determined by, the unities which they constitute. . . .

(Truly, "bodies," their attributes, their actual and potential behavior, do not *have* to be described in the differential concepts of mechanics, but remain as objective as ever they were when handled in groups, and their behavior described in integral terms covering finite time-intervals. In particular, such "integral" treatment is the only one available for teleologically-defined classes of bodies, *e.g.*, the "living." It is because the "objectivist" takes this for granted, that he wonders at finding himself *assumed* a "mechanist." None could more cordially than he concur in Kemp Smith's continuing sentences:)

> When, therefore, the extreme mechanists endeavor to view the unity of the organism as something which can be exhaustively accounted for as a resultant of the interactions of its parts, they would appear to be developing a very fruitful scientific technique into philosophical consequences which it need not imply. The behaviorists, [The reviewer pauses to observe that though he is under some obligation to let who will call him a "behaviorist," he seems never to have given anyone the least excuse for attributing to him the principal views these *behaviorists-of-literature* are said to have covered with that name. It is these behaviorists-of-literature then who] in unfaltering devotion to a strictly logical consistency, are prepared to go farther and to deny the effective existence of anything that can properly be entitled consciousness [!] (214 f.).

The reader would by this time be grateful, no doubt, for some relief from the reviewer's aggressive continuity. Let him pretend, then, a complete change of direction even though it reveal itself in the end to have been but a detour. . . .

"Discussions of value in the philosophical journals tend to lose themselves in what may appear even to careful students the most

arid controversy, the most empty and abstract argument, the most futile and unmeaning refining of distinctions based on no genuine differences" (IV; 99).

That Prall ("Present Status of the Theory of Value") after patient study of the documents should have found them of this quality, will not affront the judgment of some of their less patient readers. "But this appearance [Prall hastens to explain] does not represent their general purport"; and he feels he may "point out some really important clarifications in thought to be derived even from the present incomplete state of value theory."

Whatever one may think of the *clarification*, there can be no doubt of the *simplification* effected by a subsequent paragraph (100) which is by way of resuming the whole postulate-system of Prall's doctrine:

> The fact that valuing is not a logical process, that it is preference, and that preference is simply a reaction of the human mind in the way of liking and disliking, the fact that there is no other sort of value, and that value is thus clearly and fully irrational and in some sense direct and immediate; this fact. . . .

leads to all the rest. As Prall presents it, this is an interesting "rest"; and if we omit it from discussion it is because the postulate on which it depends is beyond our discussing. History, indeed, has given ample attention to the suggestion that *value* in one or all of its forms be made an *indefinable*. But the modern analyst well knows it is no more possible to debate the indefinability of a *detached* term than to judge the axiomatic standing of a *detached* proposition. One can only note an analyst's intention to accept "value" as one of his indefinables, and wish him better luck than those of the past have enjoyed in convincing the world of the scientific economy of a system developing out of this choice.

But this last expression, "scientific economy," brings us back to natural science and to certain motives taken by some to be value-motives having a part in its history. Of the two lucid, well-arranged, and historically-informed lectures of Lenzen on the philosophic issues of exact science, the first ("Philosophic Aspects of Relativity," IV; 135 ff.) closes with a discussion of an *a priori* element in such science. "To my mind [Lenzen writes] the best epistemological discussions of relativity have approached the subject from the Kantian point of view." Of the several cited or quoted, the author finds the position of Eddington most sympathetic and "striking": "We have found [Eddington wrote in 1921] that where science has progressed the farthest, the mind has but regained from nature that which the mind has put into nature." This is, of course, almost verbatim the Kant of the *Analytik;* (*K. d. r. V.*, A 125) but Lenzen has in mind rather the attitude of the *Dialektik*: "the principles of unity

and covariance are canons grounded in the structure of the reason. Borrowing a phrase from Kant I shall call them ideals of reason" (159 f.).

Whether or not this be the best analysis of the "ideal" mind puts into its nature-building; whether as so defined the ideal be best called "of the reason," or were better viewed as technical means (like any finite machinery) to a more ultimate "rational ideal," are questions I pass over with the greatest reluctance. For the moment there is but leisure to suggest a relation between the mind-motive "embodied" in the most exact-scientific image of nature, and certain conclusions of Adams, whose significance was previously noted for later remark.

> There is [Adams had written] a double meaning which attaches to the phrase 'habitat of mind.' Our minds are, as existences, processes which are rooted in the life process of animal bodies. In this sense our minds inhabit animal bodies. [The "objectivist," I may interpolate, would but analyze out of the body's actual and virtual relations with nature-as-a-whole our motives for calling one body mind-inhabited, another not.] Now the processes of nature may be said to live in the time-space matrix or continuum which sustains them. But in another sense nature's processes live in the physical forms and structures which these continuous processes generate and destroy. . . . Our minds (1) live in our bodies in the same way that life processes live in the chemical compounds and physical energies which sustain them; [but] (2) our minds [also] live in the significant idea systems in which the life of mind is displayed (IV; 72 f., condensed).

It will readily be imagined why the reviewer has been led to suggest a clarifying relation between the mind-motives exact science recognizes as "embodied" (the word is our own) in its nature-image, and the mind Adams sees "displayed [as] life-of-mind in significant idea systems."

> If now [Adams concludes] we mean by the habitat of mind the system of energies and structures which have engendered and which sustain the mind, then we shall say that mind inhabits [certain] animal bodies. . . . But if we mean by the habitat of mind, as well we may, the structures and objects in which the mind's processes eventuate . . . then we will say that the mind inhabits the significant objects to which it is attached through knowledge [but a creative sort of knowledge, surely?], interest, and appreciation, and that these significant structures comprise the habitat of mind (73).

Let me go no farther: I doubt I may already have forced the interpretation of my authors; for which the miles of space *ifs* take, and *buts*, must be my excuse. Suffice it, the reviewer would welcome more than another further working over of the "dialectic" (if he may venture . . .) between the mind which *as* nature "proclaims the glory of Kepler and Newton" and the mind which *in* nature once inhabited the frail fragment men called Kepler, or as might be

Newton. Whether either expression of mind have the last word of the other . . . ? It may be, as our thought of mind and nature grew more cosmic it would find the difference dividing them less chasmic.

E. A. SINGER, JR.

UNIVERSITY OF PENNSYLVANIA.

A Comparative Study of Life Ideals. The Way of Decrease and Increase with Interpretations and Illustrations from the Philosophies of the East and the West. YU-LAN FUNG. Shanghai: The Commercial Press, Ltd. 1924. Pp. xii+264.

We of the occident too often ignore the philosophies of the Far East. Our horizons are broadened and we are helped to see in better perspective the thought of our own cultural area when we realize that in regions with which we formerly had little or no intellectual intercourse the human mind has faced much the same problems that have confronted our own thinkers. Dr. Fung's little book is an attempt to state and to compare the conclusions of some of the leading philosophers of China and of the occident. Those principally dealt with are Chuang Tse, Plato, Schopenhauer, Yang Chu, Mo Ti, Descartes, Bacon, Fichte, Confucius, Aristotle, Wang Yang Ming, and Hegel. There are incidental references to several others and to Buddhism and Christianity. The author classifies philosophies in three groups —those that presuppose "that in the past man had a paradise of some sort, which he lost through his own mistake" and strive to help him regain it, those that presuppose "that if there is any paradise at all, it is a paradise *to-be-gained*, not a paradise lost" and seek to achieve it, and those that seek to reconcile these two ways of thought. The first group he calls the "nature" line of thought, and in it he places Chuang Tse, Plato, and Schopenhauer. The second, the author believes, emphasizes "art" or civilization as a means of gaining paradise, and in it are placed Yang Chu, Mo Ti, Descartes, Bacon, and Fichte. In the third, which Dr. Fung represents as seeking to find a happy mean between the two, are put Confucius, Aristotle, Wang Yang Ming, and Hegel.

Dr. Fung lays himself open to a good many criticisms. His classification is a kind of Procrustean bed and seems decidedly artificial. His title is so ambitious as to be misleading. It appears to promise a survey of at least the leading philosophies and religions the world around, but the book is almost exclusively confined to a few of the more prominent philosophers of China and the occident. India is almost entirely ignored, except for a few references to Buddhism, and such great religions as Islam, Hinduism, and Christianity, which must certainly be included in any comprehensive "comparative

study of life ideals," are either not included or are misunderstood. Christianity, the only one of the latter three that is at all dealt with, is caricatured, although probably quite unconsciously, and Jesus has apparently not been studied. Buddhism is somewhat less neglected, but is not at all adequately covered and its Mahayana developments are not considered. Even in the fields where one would expect the author to be more at home, those of Chinese philosophy, he is by no means infallible. He seems to accept without question the historicity of Lao Tse and attributes to him the composition of the Tao Teh Ching. Both of these assumptions are, to say the least, highly dubious. He dismisses with quite too scanty references the remarkable school of Sung philosophers who culminated in Chu Hsi. Readers, too, will do well to bear in mind the fundamental bias of the author. He is apparently an heir of the Confucian tradition and it is through liberal Confucian spectacles that he looks at other philosophies. He is, accordingly, interested in ethics rather than in epistemology, theology, cosmology, or cosmogeny and emphasizes the ethical aspects of philosophy. He is, too, seeking for a mean between extremes and tends to value more highly those philosophies which promise such a mean.

The book, however, has very real value. It contains a clear statement, in succinct form, of the teachings of some of the leading thinkers of China. Particularly good is the summary of Mo Ti. The occidental student familiar with China will find little that is new, but he will welcome the suggestive comparisons with western thinkers. He will, too, find stimulating the reaction of the mind of a modern Chinese student to Western philosophical thought. Those unfamiliar with China will find the summaries informing, clear, brief, and on the whole fairly accurate.

K. S. LATOURETTE.

YALE UNIVERSITY.

JOURNALS AND NEW BOOKS

ANNALEN DER PHILOSOPHIE UND PHILOSOPHISCHEN KRITIK. Band V, Heft 1. "Psychische Gestalten" und Organismen: *Hans Driesch.* Das Möglichkeitsbewusstsein und die Menschwerdung: *Julius Schultz.* Heft 2. Max Hartmanns Stellung zum vitalistischen System: *Hans Driesch.* Einheitliche Naturauffassung: *Karl Sapper.* Die Paradoxien der Mengenlehre: *W. Dieck.*

BULLETIN DE LA SOCIÉTÉ FRANÇAISE DE PHILOSOPHIE. 24e Année, No. 4. Séance du 27 Décembre, 1924. Thèse: La Crise de la Loi et du Contrat; ses manifestations; ses remèdes: *M. G. Morin.* Discus-

sion: *G. Aillet, G. Davy, H. Lévy-Bruhl, D. Parodi, F. Pécaut, E. Perrot, Ch. Rist, G. Rodrigues.*

JOURNAL DE PSYCHOLOGIE NORMALE ET PATHOLOGIQUE. XXII^e Année, No. 6. Les sentiments de joie dans l'extase: *P. Janet.* La mentalité épileptique: *H. Wallon.* Rêveurs et boudeurs morbides: *A. Borel.*

RIVISTA DI FILOSOFIA. Anno XVI, No. 2. *Comunicazioni del V Congresso italiano de filosifia di Firenze.* I postulati pedagogici: *Mariano Maresca.* Filosofia e mistica nel pensiero di Plotino: *Piero Marrucchi.* Rapporti fra mistica e pensiero: *Augusto Hermet.* Misticismo e idealismo (Riassunto): *Gino Ferretti.* Immanenza e trascendenza come antinomia filosofica: *Antonio Banfi.* Definizione dell'arte in quanto e rivelazione del sentimento: *Giulio A. Levi.* Per un grande trattato italiano di storia della filosofia: *Francesco Orestano.* I. G. B. Vico precursore della sociologia; II. Sociologia e filosofia (sunto): *Francesco Cosentini.* Ubi societas, ibi ius: *Alessandro Levi.* I. Immaturità, pericoli e danni del confondere insieme gli insegnamenti della filosofia della storia; II. Il Platonismo italiano del secolo XIX e il cosidetto idealismo tedesco: *Michelangelo Billia.* I. L'origine della dottrina pedagogica e la filosofia cristiana (sunto); II. Se la pedagogia abbia delle leggi e sia davvero scienza: *Luigi Ventura.* L'origine della filosofia ebraica medioevale: *Umberto Cassuto.* È violabile la legge morale?: *Eugenio Mingione.* Gli scritti di Mario Calderoni: *Odoardo Campa.* La teoria di Einstein e la filosofia: *Cesare L. Musatti.* La religione e la filosofia dei Valori: *Guido Della Valle.* I nuovi fondamenti dell'estetica e della filosofia: *Giuseppe Ciaccio.*

Atanassievitch, Xénia: La Doctrine Métaphysique et Geométrique de Bruno. Exposée dans son Ouvrage "De Triplici Minimo." Paris: Les Presses Universitaires de France. 1923. 157 pp.

Choisnard, Paul: Essai de Psychologie Astrale. Accompagné d'un dictionnaire de psychologie astrale, destiné à l'interpretation. Paris: Félix Alcan. 1925. 190 pp. 12 fr.

Gilson, Étienne: René Descartes. Discours de la Méthode. Texte et Commentaire. Paris: Librarie Philosophique J. Vrin. 1925. xvi + 494 pp. 35 fr.

Schlick, Moritz: Allegmeine Erkenntnislehre. (Naturwissenschaftliche Monographien und Lehrbücher. Erster Band.) 2d Edition. Berlin: Julius Springer. 1925. x + 375 pp. 18 g.m.

Stammler, Rudolph: The Theory of Justice. Translated by Isaac Husik. With appendixes by Francois Geny and John C. H. Wu. (Modern Legal Philosophy Series, edited by a Committee of the Association of American Law Schools.) New York: Macmillan Co. 1925. xli + 591 pp. $6.00.

Sticco, Maria: Il Pensiero di S. Bernardino da Siena. Milan: Società Editrice "Vita e Pensiero." 1924. viii + 202 pp.

Von Pauler, Ákos: Grundlagen der Philosophie. Berlin and Leipzig: Walter de Gruyter & Co. 1925. x + 348 pp. 12 M.

Trow, William Clark: Scientific Method in Education. (Riverside Educational Monographs, edited by Henry Suzzallo.) Boston: Houghton Mifflin Co. 1925. xi + 159 pp. $1.20.

Burt, Edwin Arthur: The Metaphysical Foundations of Modern Physical Science. An Historical and Critical Essay. (International Library of Psychology, Philosophy and Scientific Method.) New York: Harcourt, Brace & Co. London: Kegan Paul, Trench, Trubner & Co., Ltd. 1925. ix + 349 pp. $5.00.

Kretschmer, E.: Physique and Character. An Investigation of the Nature of Constitution and the Theory of Temperament. (International Library of Psychology, Philosophy and Scientific Method.) New York: Harcourt, Brace & Co. London: Kegan Paul, Trench, Trubner & Co., Ltd. 1925. xiv + 266 pp. $5.00.

NOTES AND NEWS

The program of the joint session of the Aristotelian Society, the Mind Association, and the Oxford Philosophical Society, held at Balliol College, Oxford, July 24–26, was as follows:

FRIDAY, JULY 24.

8:30 p.m. Symposium: "The Nature of Intelligence." Prof. H. Wildon Carr, Prof. A. Wolf, and Prof. C. Spearman.

SATURDAY, JULY 25.

10: a.m. Symposium: "The Concept of Energy." Mr. C. R. Morris, Dr. Dorothy Wrinch, and Prof. Leonard J. Russell.

2:30 p.m. "The Biological Basis of the Sense of Time." Dr. Ivy Mackenzie.

8:30 p.m. Symposium: "Croce's Theory of the Practical Nature of Science." Prof. J. A. Smith, Prof. A. D. Lindsay, and Dr. F. C. S. Schiller.

SUNDAY, JULY 26.

2:30 p.m. Symposium: "Plato and Aristotle." Mr. P. E. More, Prof. W. D. Ross, and Prof. G. Dawes Hicks.

8:30 p.m. Symposium: "Is Art a Form of Expression or of Apprehension?" Mr. J. MacMurray, Mr. C. E. M. Joad, and Mr. A. H. Hannay.